AMAT 220: Linear Algebra, Spring 2012 Midterm Exam, March 27

Name: ....

1] Determine the values of h and k such that the following system has: (A) no solution, (B) a unique solution, (C) infinitely many solutions. Give separate answers for each part.

$$x_1 + 3x_2 = k$$
$$4x_1 + hx_2 = 8$$

2] Solve the following system and then write the solution in parametric vector form.

$$\begin{aligned} 2x_1 + 2x_2 + 4x_3 &= 8 \\ -4x_1 - 4x_2 - 8x_3 &= -16 \\ -3x_2 - 3x_3 &= 12 \end{aligned}$$

3] Determine whether the following sets of vectors in $\mathbb{R}^3$ are linearly independent. Justify each answer.

- $\begin{bmatrix} 1 \\ -9 \\ 6 \end{bmatrix}, \begin{bmatrix} -\frac{1}{3} \\ 3 \\ -2 \end{bmatrix}$

- $\begin{bmatrix} 11 \\ -13 \\ 17 \end{bmatrix}, \begin{bmatrix} 0 \\ 26 \\ -34 \end{bmatrix}$

- $\begin{bmatrix} 5 \\ -7 \\ 9 \end{bmatrix}$

- $\begin{bmatrix} 5 \\ -7 \\ 9 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

- $\begin{bmatrix} 5 \\ -7 \\ 9 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- $\begin{bmatrix} 5 \\ -7 \\ 9 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -7 \\ 5 \end{bmatrix}$

4] • How many rows and columns must a matrix A have in order to define a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^5$ by the rule $T(\mathbf{x}) = A\,\mathbf{x}$?

$$\text{Rows} = \;\ldots\ldots\ldots$$

$$\text{Columns} = \;\ldots\ldots\ldots$$

• Find the standard matrix of the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ such that:

$$T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}3\\1\\-3\\-1\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\2\\2\\0\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}.$$

• Find the standard matrix of the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ such that:

$$T\left(\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}\right) = \begin{bmatrix}x_1 - x_2 + x_3\\-3x_2 - 7x_3\end{bmatrix}.$$

5] Find the inverses of the following matrices, if they exist.

- $\begin{bmatrix} 1 & -3 \\ 4 & -9 \end{bmatrix}$

- $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

6] Combine the methods of row reduction and cofactor expansion to compute the following determinants.

- $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 6 & 2 \\ 0 & -3 & 1 \end{vmatrix}$

- $\begin{vmatrix} 2 & 5 & -3 & -1 \\ 3 & 0 & 1 & -3 \\ -6 & 0 & -4 & 9 \\ 4 & 10 & -4 & -1 \end{vmatrix}$

7] If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$, compute the following determinants:

- $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$

- $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix}$

- $\begin{vmatrix} a & b & c \\ d & e & f \\ -2g & -2h & -2i \end{vmatrix}$

- $\begin{vmatrix} a+2g & b+2h & c+2i \\ d & e & f \\ g & h & i \end{vmatrix}$

- $\begin{vmatrix} 2a+g & 2b+h & 2c+i \\ d & e & f \\ g & h & i \end{vmatrix}$

- $\begin{vmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{vmatrix}$

- $\begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix}$

- $\begin{vmatrix} c & b & a \\ f & e & d \\ i & h & g \end{vmatrix}$

8] If A and B are 4×4 matrices with $\det A = -2$ and $\det B = 5$, compute the following determinants:

- $\det (AB)^{-1}$
- $\det (AB)^T$
- $\det A^T B$
- $\det 3A$
- $\det B^{-1}$
- $\det A^3$
- $\det B^{-1}AB$
- $\det B^{-1}A$